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Hierarchical Scales and "Family" of Black Holes

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An attempt to unify the scales of physical quantities is presented within the framework of a hypothesis about the existence of a family of black holes. There exists a scale characterized by parameters $m_X \sim 4 \times 10^{14}$ GeV and $\alpha_{\text{GUM}} \sim 1/25$ which may be identified with a scale for the grand unification.

In contemporary physics some hierarchical scales for the physical quantities (lengths, energies, times, etc.) are observed. For example, atomic and molecular processes occur in the domain characterized by length $O(10^{-8})$ cm, but electromagnetic: $O(10^{-10}-10^{-11})$ cm. The length $O(10^{-13})$ cm is just a typical distance for nuclear physics. The majority of physicists believe that weak and quantum-gravitational effects will be manifested at distances of the order of $l_w \sim 6.7 \times 10^{-17}$ cm and $l_g \sim 8.0 \times 10^{-33}$ cm, respectively.[1] Some models (Pati and Salam, 1973; Georgi and Glashow, 1974; Buras et al., 1978; Goldman and Ross, 1979; Marciano, 1979) suggest that grand unification seems to take place at the energy scale of order $m_X \sim 6.10^{14}$ GeV [at length $O(10^{-29})$ cm] with the gauge coupling constant $\alpha_{\text{GUM}} = g^2/4\pi \sim 1/40$ at this scale.

The present note is devoted to obtaining the parameters of m_X and α_{GUM} for the grand unification theories within the framework of a hypothesis about the existence of a family of the black holes. It appears that because of this hypothesis all above-mentioned typical lengths are confined to the unique scale measured by a parameter $\alpha/2$, α being the fine-structure constant. Following an idea of Tennakone (1974) we make the assumption that some elementary particles are black holes (or monopoles) in the strong

[1]The connection between $l_g[l_w]$ and the Newton constant G_g (the Fermi constant G_F) is $l_g = (8\pi\hbar G_g/c^3)^{1/2}$ $[l_w = (G_F/\hbar c)^{1/2}]$.

gravitational field. Let us consider the "family" of the chain of black holes

$$\begin{pmatrix} f_0 \\ f_1 \end{pmatrix}, \quad \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}, \quad \begin{pmatrix} f_2 \\ f_3 \end{pmatrix}, \ldots$$
$$i=1, \qquad 2, \qquad 3, \ldots \tag{1}$$

(it is assumed that $m_{f_{i-1}} \ll m_{f_i}$, $i=1,2,\ldots$) which are singularities in the Nordström–Reissner solutions of the Einstein equation for a particle of mass m_{f_i} and charge e. This solution is given by the metric

$$ds^2 = -\gamma^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta \, d\phi^2 + \gamma \, dt^2 \tag{2}$$

where

$$\gamma = 1 - \frac{2G_g m_{f_i}}{c^2 r} + \frac{G_g e^2}{c^4 r^2} \tag{3}$$

with G_g the Newtonian gravitational constant.

If now we assume that the structure of space-time in the immediate neighborhood of the charged heavy particles f_i belonging to an ith pair of the family (1) is determined by strong gravity, then the constant G_g in (3) should be replaced by the strong gravitational constant G_s^i:

$$G_s^i m_{f_i}^2 / \hbar c \sim 1$$

Then the singularities in the strong gravitational field of the charged particle f_i are given by the metric (2) with

$$\gamma = \gamma_s = 1 - \frac{2G_s^i m_{f_i^0}}{c^2 r} + \frac{G_s^i e^2}{c^4 r^2} \tag{4}$$

where $m_{f_i^0}$ is the mass of an uncharged partner of the heavy particle f_i which is assumed to exist too.

Since $G_s^i m_{f_i}^2 / e^2 \gg 1$, the equation $\gamma_s = 0$ has two real roots $r_{f_{i-1}}$ and r_{f_i}. For those one may easily obtain the following ratio (for details, see Tennakone, 1974):

$$m_{f_{i-1}} / m_{f_i} = r_{f_{i-1}} / r_{f_i} \tag{5}$$

or

$$\frac{2G_s^i m_{f_{i-1}}}{c^2 r_{f_{i-1}}}=1, \qquad \frac{2G_s^i m_{f_i}}{c^2 r_{f_i}}=1 \tag{6}$$

where

$$r_{f_{i-1}}=e^2/2m_{f_i^0}c^2 \tag{7}$$

Here r_{f_i} may be interpreted as the Schwarzschild radius of the singularity corresponding to particle f_i, m_{f_i} and $m_{f_{i-1}}$ are masses of heavy and light particles (black holes) belonging to an ith pair of (1), respectively.

Notice that the strong gravitational constant G_s^i comes out as a by-product of the theory. From (6) and (7) we have

$$G_s^i=e^2/4m_{f_{i-1}}m_{f_i^0} \tag{8}$$

Now we determine a parameter which characterizes the hierarchical scales for the physical quantities in black holes physics. Due to Tennakone (1974), identifying f_0 with the electron and f_1 with the proton ($f_1^0 \equiv$neutron) we get from (5) (since $m_p \approx m_n$)

$$r_1=\frac{e^2}{2\hbar c}\cdot\frac{\hbar}{m_e c}=\frac{\alpha}{2}\cdot\frac{\hbar}{m_e c}=1.4\times 10^{-13}\,\text{cm}$$

or in terms of the energy scale

$$m_{w_1}=\hbar/r_1 c=140 \text{ MeV}$$

Further, assuming $f_1 \equiv p$ in the next pair ($i=2$) of the family (1), we have

$$m_p=\frac{e^2}{2m_{f_2^0}c^2}\frac{m_{f_2}}{r_2}$$

Since $m_{f_2} \approx m_{f_2^0}$. Then

$$r_2=\frac{\alpha}{2}\frac{\hbar}{m_p c}=7.7\times 10^{-17}\,\text{cm} \qquad \text{or} \qquad m_{w_2}=\frac{\hbar}{r_2 c}=274 m_p=257\,\text{GeV}$$

etc.

We see that the number $\alpha/2$ plays the role of a scale in the transition from one pair to another in the family of black holes, and it may be interpreted as a parameter of hierarchy in physics. Further it is postulated that the mass ratio of each hole belonging to the family (1) is given by the parameter scale $\alpha/2$:

$$m_{f_i}=\frac{2}{\alpha}m_{f_{i-1}} \qquad (i=2,3,\ldots) \tag{9}$$

or

$$m_{f_i}=\left[\frac{2}{\alpha}\right]^{i-1}m_p$$

Typical scales for $r_i(m_{w_i})$ corresponding to this parameter $\alpha/2$ are shown in Table I. Now with the same scale $\alpha/2$ and formally continuing hierarchical scales shown in Table I, to the left we obtain typical lengths 3.86×10^{-11} cm and 1.06×10^{-8} cm for electromagnetic and atomic–molecular processes. Further continuation of hierarchical scales to the right is impossible and the chain of black holes breaks at $i=9$. Since in the case $i>9$ (we obtain an object with mass larger than the Planck mass $m_{Pl}\sim1.2\times10^{19}$ GeV) classical solutions of the Einstein equations are not acceptable and quantum gravitational effects should be taken into account.

Let us calculate the gauge coupling constant at the scale of order $m_X\sim4\times10^{14}$ GeV. For this assuming $m_{f_7^0}\sim m_{f_7}$ and taking into account (9), from (8) we get

$$\alpha_{\text{GUM}}=\frac{g^2}{4\pi\hbar c}=\frac{G_s^7m_{f_7}^2}{4\pi\hbar c}=\frac{\alpha}{16\pi}\left(\frac{m_{f_7}}{m_{f_6}}\right)\sim\frac{1}{25}$$

From Table I we see that

$$m_X/m_W\sim1.5\times10^{12}$$

So, in conclusion, we notice that the scale corresponding to the seventh pair of black holes may claim to be a scale of the grand unification (or, at least, a lower bound to the grand unification scale). In distinction to the paper of Tennakone where one pair (e,p) of black holes was considered, in this note we have assumed that there exists a family of black holes in the strong gravitation field. Within the framework of this hypothesis the number of pairs of black holes is restricted and equals $9=1+8$.

TABLE I

Black holes	$\begin{pmatrix} e \\ p \end{pmatrix}$	$\begin{pmatrix} p \\ f_2 \end{pmatrix}$	$\begin{pmatrix} f_2 \\ f_3 \end{pmatrix}$	$\begin{pmatrix} f_3 \\ f_4 \end{pmatrix}$	$\begin{pmatrix} f_4 \\ f_5 \end{pmatrix}$	$\begin{pmatrix} f_5 \\ f_6 \end{pmatrix}$	$\begin{pmatrix} f_6 \\ f_7 \end{pmatrix}$	$\begin{pmatrix} f_7 \\ f_8 \end{pmatrix}$	$\begin{pmatrix} f_8 \\ f_9 \end{pmatrix}$
No. (i)	1	2	3	4	5	6	7	8	9
r_i (cm)	1.4×10^{-13}	7.7×10^{-17}	2.8×10^{-19}	1.0×10^{-21}	3.7×10^{-24}	1.3×10^{-26}	4.7×10^{-29}	1.7×10^{-31}	6.6×10^{-33}
m_{w_i} (GeV)	0.14	257	7.0×10^{4}	1.9×10^{7}	5.8×10^{9}	1.4×10^{12}	3.9×10^{14}	1.0×10^{17}	2.8×10^{19}
Comparison	m_π	m_W	m_{mon}				m_X		m_{Pl}

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